

# ANALYSIS OF DECISION THEORETIC MODULATION CLASSIFICATION METHODS FOR DIGITAL COMMUNICATION SIGNALS

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## ABSTRACT

Automated modulation classification is a fundamental requirement for electronic support measures. Existing automated classifiers use a variety of different modulation recognition techniques. This paper reviews the category of decision-theoretic approaches and discusses the relationships between decision-theoretic methods and other statistical modulation classification methods.

## 1. INTRODUCTION

The goal of modulation classification is to take an unknown signal and identify the modulation type with a high probability of success using a short observation time. Many automated modulation classification (AMC) algorithms have been developed during the last two decades and surveys of these were presented in (Su and Kosinski, 2002, 2003). The two commonly recognized approaches to AMC are 'Decision-Theoretic' (DT) and 'Pattern Recognition' (PR) (Huang and Polydoros, 1995). The decision-theoretic approach provides an optimal solution in the sense that it minimizes the probability of false classification if all assumptions are met. Within the DT approach, AMC is formulated as a multiple composite hypothesis-testing problem, and the hypothesis is resolved using maximum likelihood techniques. Various implementations of maximum likelihood have been proposed for DT-AMC based upon different assumptions regarding the unknown signal. An understanding of these differences is critical to proper selection of an AMC technique for a particular scenario.

## 2. DT ALGORITHM OVERVIEW

Three different decision-theoretic algorithms have been found in the literature: average likelihood ratio test (ALRT), generalized likelihood ratio test (GLRT), and hybrid likelihood ratio test (HLRT). The ALRT is a popular AMC approach applied to PSK and QAM modulation types. ALRT (Huang and Polydoros, 1995; Sills, 1999; Wei and Mendel, 2000; Hong and Ho, 2003; Beidas and Weber, 1995; Beidas and Weber, 1998, El-Mahdy and Namazi, 2002) treats unknown parameters as random variables (RV's) and the likelihood function is computed by averaging over them. This requires a hypothesis for the probability density functions (pdfs) of the RV's. If the true pdfs coincide with the hypotheses, then the results are optimal. The ALRT is

computationally intensive but current microprocessors have made the ALRT practicable. In general, the performance of ALRT AMC is very sensitive to modulation parameters such as symbol timing, baud rate, carrier frequency, carrier phase, pulse shape, and noise power. The ALRT is also affected by channel fading and the *type* of noise that is present.

The GLRT (Panagiotou et al., 2000) treats the candidates as unknown deterministic values and the maximum likelihood test is applied as if the true values were known. The HLRT (Panagiotou et al., 2000) is a hybrid approach that treats some of the candidate parameters as random variables with known pdfs and some of the candidate parameters as unknown deterministic variables.

The common deficiencies of all likelihood-ratio test approaches are that the assumptions are restrictive, the pre-processing is intensive, and the hypothesis of "unknown type" is not included.

## 3. RELATIONSHIP TO STATISTICAL TESTS

'Statistical tests' based on the mean, variance, and histogram have been applied to AMC. It is important to understand properly the relationship of these to other AMC approaches.

To begin, consider the ALRT for classifying PSK/QAM modulation signals. For the  $i$ -th hypothesis  $H_i$ , the joint log-likelihood function is (Wei and Mendel, 2000):

$$L(H_i | r_K) = \sum_{k=1}^K T^{(i)}(k), \quad (1)$$

where

$$T^{(i)}(k) = \ln \left\{ \frac{1}{M_i} \sum_{j=1}^{M_i} \exp \left\{ -\frac{\|r(k) - b^{(i)}(j)\|^2}{2\sigma^2} \right\} \right\}, \quad (2)$$

and where  $r(k)$  is a symbol-based complex data series of length  $K$ , preprocessed from the signal emitted from a non-cooperative transmitter through an AWGN channel with a two-sided power spectral density of  $\sigma^2$ , and  $b^{(i)}(j)$ ,  $j=1, 2, \dots, M_i$  is a complex number and is the  $j$ -th reference state of  $i$ -th modulation type. The decision of modulation classification is made based on the criterion:

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choose  $1 \leq I \leq M_i$  as the modulation type if  $L(H_i | r_k)$  is a maximum.

Consider now the histogram test. This test is very popular in AMC practice for classifying real variables, such as modulation phase, frequencies, or amplitudes (Liedtke, 1984; Hsue and Soliman, 1989). The histogram is constructed from a density table with the intervals shown on the  $x$ -axis and the number of occurrences in each interval represented by the height of a rectangle located above the interval. To determine the similarity between the ALRT and the histogram test, we choose both  $r(k)$  and  $b^{(i)}(j)$  as real values. A density table is constructed by dividing  $T^{(i)}(k)$  into  $Q$  equally divided intervals denoted by  $T_1^{(i)}, T_2^{(i)}, \dots, T_Q^{(i)}$ , and counting the number of  $T^{(i)}(k)$ s occupying the  $q^{th}$  interval, denoted by  $D_q^{(i)}$ , for  $q = 1, 2, \dots, Q$ . If  $T^{(i)}(k)$  is bounded by  $\{-R_n^{(i)}, R_p^{(i)}\}$ , i.e.,  $-R_n^{(i)} \leq T^{(i)}(k) \leq R_p^{(i)}$  for all  $k$ , we find

$$T_q^{(i)} = \frac{I_q^{(i)} + I_{q+1}^{(i)}}{2} \quad \text{if } I_q^{(i)} \leq T^{(i)}(k) < I_{q+1}^{(i)}, \quad (3)$$

where

$$I_q^{(i)} = -R_n^{(i)} + \frac{R_p^{(i)} + R_n^{(i)}}{Q}(q-1). \quad (4)$$

Therefore the quantized version of (1) will be

$$L(H_i | r_k) = \sum_{q=1}^Q T_q^{(i)} D_q^{(i)}. \quad (5)$$

Notice that the data series  $D_q^{(i)}$  is the histogram data of  $r(k)$ , for  $k=1, 2, \dots, K$ , with  $Q$  bins, and  $T_q^{(i)}$  is the template associated with  $H_i$ . In the limit as  $Q \rightarrow K$ , the results of the histogram test approach the results of the ALRT, showing that the histogram test is a special case of the ALRT. It is remarkable also that (1)-(5) provide asymptotic optimal templates for histogram test.

The variance test is another frequently used approach in AMC. Usually, it is used for coarse estimation of modulation features such the separation of FSK from PSK. To determine the relationship of the variance test and the ALRT, we choose both  $r(k)$  and  $b^{(i)}$  as real variables and define  $b^{(i)} = b$  as a single reference state. Therefore, (2) can be simplified to

$$T^{(i)}(k) = \ln \left\{ \exp \left\{ -\frac{\|r(k) - b^{(i)}\|^2}{2\sigma^2} \right\} \right\} = -\frac{(r(k) - b^{(i)})^2}{2\sigma^2} \quad (6)$$

and the joint log-likelihood function in (1) becomes

$$L(H_i | r_k) = -\frac{1}{2} \sum_{k=1}^K X_i^2(k) \quad (7)$$

where  $X_k = \frac{r(k) - b}{\sigma}$ . Since there is only one state in the variance test, the probability distribution function is not required in (7). Equivalently, type  $I$  will be chosen if  $\sum_{k=1}^K X_i^2(k)$  is a minimum. Thus we determine that the variance test is also a special case of the ALRT.

## CONCLUSION

The ALRT is an effective approach in DT-AMC if the unknown signal can be pre-processed to satisfy the required assumptions. The histogram and variance approaches have been determined to be special cases of the ALRT.

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